

Quark Dynamics on Phase-Space

A. Bonasera*

*Laboratorio Nazionale del Sud, Istituto Nazionale di Fisica Nucleare, Via S. Sofia 44, 95123
Catania, Italy*

Abstract

We discuss the dynamics of quarks within a Vlasov approach. We use an interquark (qq) potential consistent with the indications of Lattice QCD calculations and containing a Coulomb term, a confining part and a spin dependent term. Hadrons masses are shown to arise from the interplay of these three terms plus the Fermi motion and the finite masses of the quarks. The approach gives a lower and an upper bound for hadrons. The theoretical predictions are shown to be in fairly good agreement with the experimental data.

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*bonasera@lns.infn.it

Very recently the approval and construction of new machines capable of accelerating light and heavy ions at 100 GeV/nucleon and more has stimulated the interest for the search of a quark gluon plasma [1]. Methods developed to describe the dynamics of two colliding heavy ions at low energies [2] can be very useful in this new area when modified to take into account the intrinsic structure of the hadrons. As an example we discuss in this paper an approach based on the Vlasov equation (VE). This equation can be obtained from the quantum Bogoliubov-Born-Green-Kirkwood-Yvon(BBGKY) hierarchy by means of the Wigner transform in the limit $\hbar \rightarrow 0$ [2,3]. It can include quantum effects such as the Pauli principle and can be easily extended to relativistic dynamics [1,2]. In this paper we will apply the VE to the dynamics of the constituents quarks in hadrons. The quark masses can be quite small as in the case of (u,d,s) quarks, (see table I for the values used in this paper), so that a relativistic treatment is necessary. As far as the interaction is concerned we will start from a two body potential consistent with Lattice Quantum Chromo Dynamics (LQCD) calculations [1,4], which displays both a Coulomb $U(r \rightarrow 0) \propto 1/r$ behaviour and a confining part $U(r \rightarrow \infty) \rightarrow \infty$. In particular, we will use the Richardson's potential which depends on the scale parameter Λ [5]. From such a starting point we calculate the time evolution of the quarks and the masses of the hadrons. The general experimental features are quite reasonably reproduced, even though the radii of light hadrons are underestimated. We notice that a similar approach has been proposed in [6], but a different interaction was used.

We briefly recall some general features of the VE, interested people should look the (not complete) list of references for more details on this equation [1-3]. The VE gives the time evolution of the one body distribution function $f_{qc}(r, p, t)$ in phase-space:

$$\partial_t f_{qc} + \frac{\vec{p}}{E} \cdot \nabla_r f_{qc} - \nabla_r U \cdot \nabla_p f_{qc} = 0 \quad (1)$$

where $U(r)$ is the potential (discussed below) which governs the quarks dynamics, $E = \sqrt{p^2 + m^2}$ is the energy and m is the quark mass. The underscripts indicate that the distribution function depends on the flavor (q) and color (c) of the quarks. Notice that the kinetic part is properly relativistically treated, while the potential term is non relativistic. A relativistic extension of the Richardson's potential is discussed in [7] but will not be included here.

Numerically the VE equation is solved by writing the one body distribution function as:

$$f_{qc}(r, p, t) = \frac{1}{n_{tp}} \sum_i^N g_r(r - r_i(t)) g_p(p - p_i(t)) \quad (2)$$

where the g_r and g_p are sharply peaked distributions (such as delta functions, gaussian or other simple functions), that we shall treat as delta functions. $N = Q n_{tp}$ is the number of such terms, $Q = q + \bar{q}$ is the total number of quarks and antiquarks (for a meson $Q=2$). Actually, N is much larger than the total quark number Q , so that we can say that each quark is represented by n_{TP} terms called test particles(tp). The rigorous mean field limit can be obtained for $n_{TP} \rightarrow \infty$ where the calculations are of course numerically impossible, even though the numerical results converge rather quickly. Inserting eq.(2) in the Vlasov equation gives the Hamilton equations of motions for the t.p. [2]:

$$\begin{aligned}\dot{\mathbf{r}}_i &= \frac{\mathbf{p}_i}{E_i} \\ \dot{\mathbf{p}}_i &= -\nabla_{\mathbf{r}_i} U,\end{aligned}\tag{3}$$

for $i = 1 \dots N$. The total number of tp used in this work ranges from 5000 to 50000 with no appreciable change in the results. The equations of motion (3) are solved by using a $O(\delta t^4)$ Adams-Bashfort method [8].

Let us now specify the $q\bar{q}$ -potential. In agreement to LQCD calculations [4,5] we have for mesons ($\hbar = 1$):

$$U(r) = \frac{8\pi}{33 - 2n_f} \Lambda \left(\Lambda r - \frac{f(\Lambda r)}{\Lambda r} \right) + \frac{8\pi}{9} \bar{\alpha}_s \frac{\langle \sigma_q \sigma_{\bar{q}} \rangle}{m_q m_{\bar{q}}} \delta(\mathbf{r})\tag{4}$$

where

$$f(t) = 1 - 4 \int \frac{dq}{q} \frac{e^{-qt}}{[\ln(q^2 - 1)]^2 + \pi^2}\tag{5}$$

n_f is the number of quark flavors involved and the parameter Λ has been fixed to reproduce the masses of heavy $c\bar{c}$ and $b\bar{b}$ systems in [5]. In eq.(4) we have added to the Richardson's potential the chromomagnetic term, very important to explain the masses of different resonances for light quarks. In this work the expectation value of $\langle \sigma_q \sigma_{\bar{q}} \rangle$ is used depending on the relative spin orientations of the constituent quarks. For instance for the pion this term is equal to -3 while for a ρ meson it is equal to +1 [4]. The δ function is approximated to a gaussian i.e. we make the replacement $\delta(\mathbf{r}) \rightarrow \frac{1}{(2\pi\sigma^2)^{3/2}} e^{-\frac{r^2}{2\sigma^2}}$, and we fixed $\sigma = 0.5 fm$. The results depend on the ratio of the average value of the strong coupling constant $\bar{\alpha}_s$ to the variance of the gaussian. Such ratio was fixed to fit the mass difference between π and ρ mesons (see below). Clearly the chromomagnetic interaction becomes unimportant for very heavy quarks. The potential term eq.(4) acts between two quarks, since we are describing quarks as a swarm of tp, we normalize the potential by a factor $1/n_{tp}$ [9].

The initial conditions are given by randomly distributing the tp in a sphere of radius r in coordinate space and p_f in momentum space. p_f is the Fermi momentum estimated in a simple Fermi gas model by imposing that a cell in phase space of size $h = 2\pi$ can accommodate at most two identical quarks of different spins. A simple estimate gives the following relation between the quarks density n_q and the Fermi momentum:

$$n_q = \frac{g_q}{6\pi^2} p_f^3\tag{6}$$

an analogous formula can be derived for \bar{q} [1]. The degeneracy number $g_q = n_c \times n_s \times n_f$, where n_c is the number of colors and n_s is the number of spins [1]. For quarks and antiquarks 3 different colors are used red, green and blue (r,g,b) [4]. From the above equation we see that the Fermi momentum for quarks distributed in a sphere of radius 0.5 fm is of the order of 0.5 GeV/c. Thus relativistic effects become important for quark masses less than 1 GeV. For instance for the π case discussed below, if we calculate the total energy of the system relativistically we obtain 0.14 GeV, while using the nonrelativistic limit we obtain about 1

GeV! We stress that since the system is properly antisymmetrized at time $t=0\text{fm}/c$, it will remain so at all times since the VE conserves the volume in phase-space [2].

The masses of the hadrons are determined by finding the minimum total energy of the system as a function of the initial radius r . For each initial radius r a Fermi momentum is deduced from eq.(6) and this gives (after adding the potential term) a total energy of the system which is of course constant in time. Changing the initial radius changes the total energy, and it has a minimum at $r = \bar{r}$. Thus it is the interplay among the Fermi motion, the potential term and the quark masses which determines the masses of the hadrons.

In order to test our approach, we have first studied heavy quark systems using the same values for the c and b masses, $n_f = 3$ and $\Lambda = 0.398\text{GeV}$ as in ref. [5] and neglecting the chromomagnetic term. We obtained the minimum mass values of 2.9 and 9.5 GeV for the $c\bar{c}$ and $b\bar{b}$ systems, to be contrasted with 3.1 GeV and 9.4 GeV obtained in [5]. The good agreement to the quantum calculation of [5] suggests that the VE is quite well justified and in particular the mean field approximation is good. This is probably due to the fact that the interparticle potential is the result of many gluons exchanges and that the hadrons are made of a combination of colored quarks and in this sense the mean field approximation is reasonable. Also, it is important to stress that in a quantum calculation the bare two body potential is folded with smooth $q\bar{q}$ wave-functions. The smoothness of the wave functions is simulated in the VE with the use of (a large number of) tp.

In order to study the limits and merits of our approach we have extended the calculations to lighter quarks. For such systems the chromomagnetic term is important to reproduce the experimental values of the masses. Using the same value of the scale parameter $\Lambda = 0.398\text{GeV}$ as in [5] and fixing the (u,d) quark masses and the strong coupling constant $\bar{\alpha}_s$, we can easily reproduce the pion mass, but not the ρ mass at the same time. Thus we have readjusted the values of the scale constant and the quark masses to reproduce the data. We found a good fit for mesons by using $\Lambda = 0.250\text{GeV}$, $\bar{\alpha}_s = 0.225$ [4] and the quark mass values given in Table I. We notice that the parameters and the heavy quark mass values are in good agreement with currently accepted ones [4,10]. The masses of (u,d) quarks in table I is somewhat smaller than the 300 MeV used in many potential models [4]. This is due to the fact that these models are non relativistic but, as we stressed above, because of the Fermi motion, relativistic effects are quite important. In the relativistic approach of [7], where the Richardson's potential was used as well, the (u,d,s) quark masses have values comparable to ours. The small differences between our results and [7] are most probably due to their relativistic generalization of the Richardson's potential.

The parameters entering our model are essentially fixed on some meson masses. As a consistency check we extended the calculations to the baryons case with the usual modifications of a factor 1/2 to the potential eq.(4), as suggested by LQCD considerations [4].

From the knowledge of the distribution function at each time step, we can easily calculate the density and the potential of the system at one time step. An average over time of these quantities can be performed as well. In the calculations the time averages were performed over a time interval up to 100 fm/c. In figure (1), we plot the density (left column) as a function of the distance r from the center of the system and obtained as average over the one body distribution function at one time step(square symbols), and over time as well (full line). The top panel correspond to a total energy of a (u,d) system of 0.140 GeV and a root mean square (RMS) radius (averaged over time) of 0.33 fm, i.e. the pion. We stress that

this is the minimum for the total energy obtained when the total quarks spin add to $S=0$, i.e. a pseudoscalar mesons [4]. The middle panel corresponds to a total energy of 0.85 GeV and a RMS of .53 fm for a (uud) system, i.e. a nucleon. A similar calculation but for $S=3/2$ (Δ) gives a mass of 1.3 GeV and a RMS of 0.75 fm (bottom panel in figure 1). In all cases, time and tp averages are identical which implies that the systems are in equilibrium. The densities obtained averaging over the distribution function at one time step, do not extend to very low values because of the finite number of tp. The rms radius of the π and n are smaller than data [4], however the densities extend much further than the rms radius, and they fall off exponentially similarly to experiments. For increasing quark masses the rms becomes more reasonable as compared to data or other calculations.

The average potential can be easily estimated as [9]:

$$\bar{U}(r) = \frac{1}{n_{tp}} \sum_j U(r, r_j) \quad (7)$$

The average potential displays some interesting features, see fig.1(right column). First, all the potentials go to zero for all systems (even though it is explicitly shown in the figure for the Δ -case only), and for large distances in contrast to the bare potential eq.(4) that diverges linearly, just because the densities go to zero. The average potential for π and n is an attractive pocket, and the confining term gives some contribution at large distances. For the Δ case, the chromomagnetic term is repulsive thus the RMS of the system is larger than n and the confining term only is responsible for keeping the system bound. Looking at the Δ -density we see that the repulsion at small distances gives a smaller and flatter density as compared to the n case.

In figure (2) we display the calculated (open symbols) mass of the resonances vs. the sum of the quark masses (cfr. table I), for mesons (top) and baryons (bottom). The circle symbols refer to attractive, while the squares to repulsive chromomagnetic term in eq.(4) [4]. The corresponding experimental data [10] are given by the full symbols. The overall agreement is quite good in all cases and some predictions for resonances not yet observed are also given.

In order to understand how other higher resonances appear for fixed quark types, we define a dipole operator for mesons, analogous to the nuclear case [2,11]:

$$D(t) = \sum_q p_z(t) - \sum_{\bar{q}} p_z(t) \quad (8)$$

where the sum is extended over all the tp for q and \bar{q} , the choice of the z-axis is of course arbitrary. Resonances are better seen by defining the dipole strength function $S(E)$, where E is the energy:

$$S(E) = |F(E)|^2 \quad (9)$$

with the Fourier transform

$$F(E) = \int dt [e^{i(Et/\hbar)} D(t)] \quad (10)$$

In figure 3 we plot the function $S(E)$ vs E for the π (ud-quarks) case. Strong resonances extend to about 1. GeV. This implies that we can have mesons up to 1. GeV above the

ground state mass. In the case of the heavier $b\bar{b}$ quarks, resonances extend to about 0.5 GeV above the ground state. Experimentally resonances are seen for instance for the $b\bar{b}$ case to about 2 GeV above the smallest resonance [10]. In our model we find that to obtain resonances at higher energies we need to increase the strength of the confining term. Thus the measured upper values of the hadron masses can give a constraint on the value of the string tension.

The results discussed above prove that the VE is suitable to describe the quarks dynamics in hadrons despite the simple form for the two body potential. The approach works rather well for heavy quarks, in that it is able to reproduce the masses and radii of the heavy hadrons (as compared to data or other calculations [4,5]) and also the masses of light hadrons. The radii of light quarks systems are underestimated which could be a hint for relativistic corrections to the potential term as discussed for instance in [7].

The method is quite easy to implement and the numeric is rather well under control. The equation of state of quark matter can be calculated within the same formalism. Future work will be also to implement a collision term which should help to understand the dynamics of colliding hadrons.

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Table I

Quark masses used in this work.

<i>QuarkMass</i>	GeV
<i>u</i>	0.13
<i>d</i>	0.13
<i>s</i>	0.35
<i>c</i>	1.45
<i>b</i>	4.8
<i>t</i>	180.

FIGURES

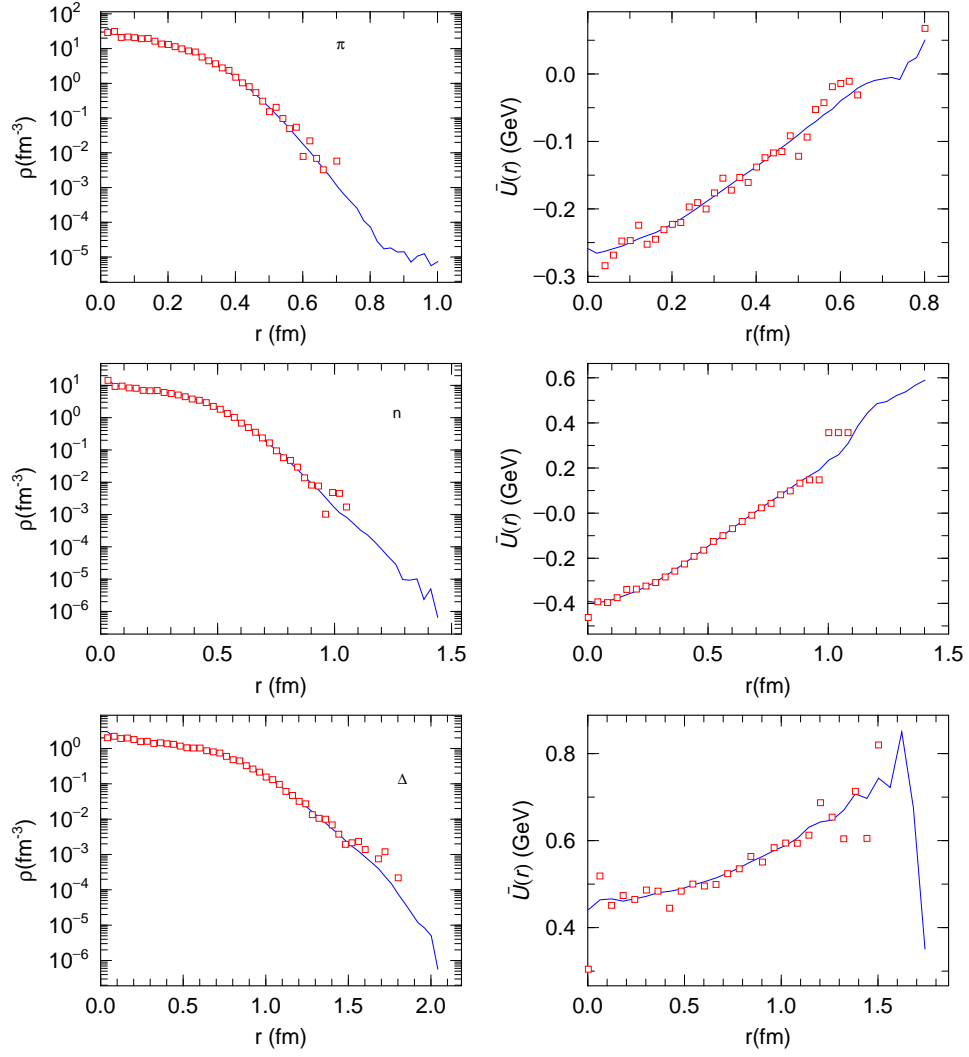


FIG. 1. Average density (left column) for a π (top), n (middle) and a Δ (bottom) hadrons. The average is over tp (squares) and over tp and time (full line). Similarly for the average potential (right column).

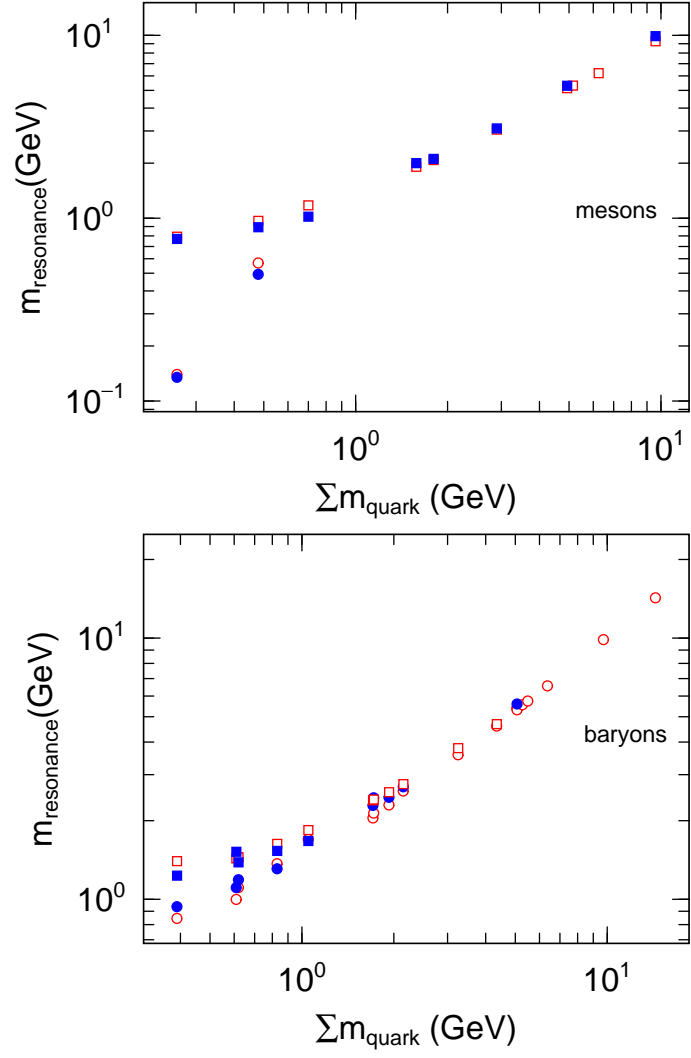


FIG. 2. Meson masses (top) vs. masses of the $q\bar{q}$ pair. The symbols refer to the observed (full symbols) and calculated (open symbols) masses. The circles refer to the pseudoscalar and the squares to vector mesons masses. Similarly for baryons (bottom). Data are taken from [9].

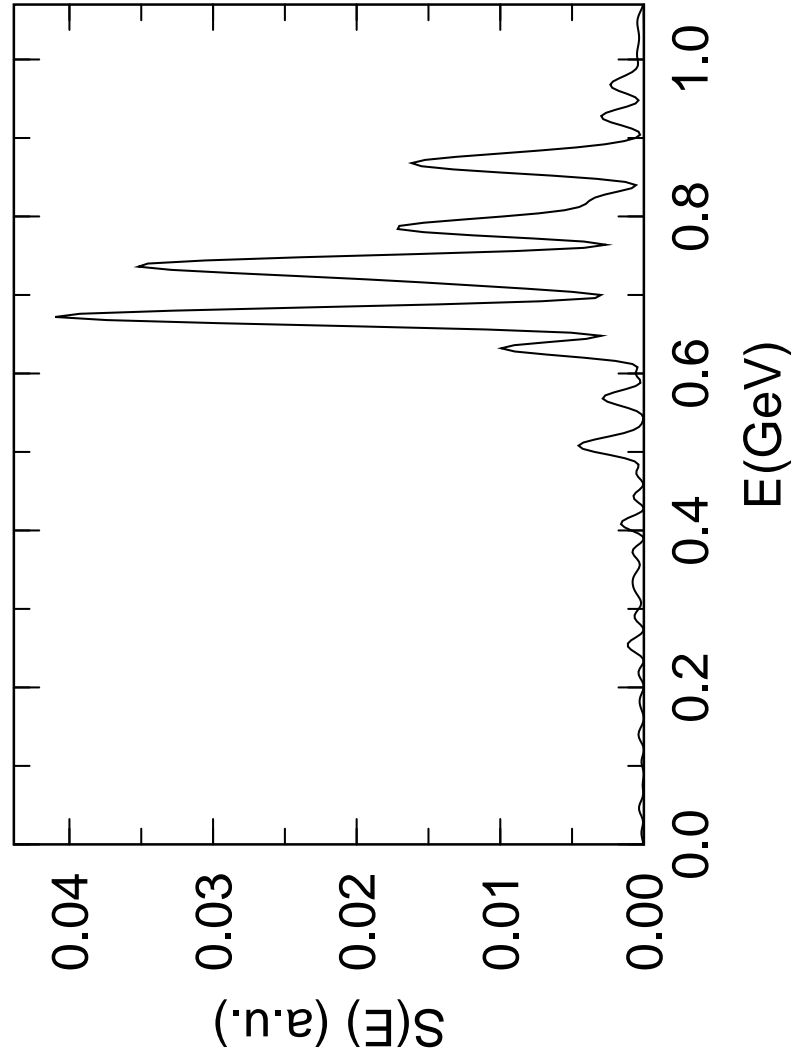


FIG. 3. Strength function (in arbitrary units) versus energy for the π case.